REVISITING MATHEMATICAL ATTITUDES OF STUDENTS IN SECONDARY EDUCATION

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The aim of this paper is three-fold. First, we review and briefly synthesise the main points of the recent debate around the concept of 'attitudes to mathematics'. We then present the measurement methodology we employed to capture 'attitudes to mathematics' in the context of a large scale UK project with secondary school students, and how these results inform the theoretical debate. Finally, we report some substantive results about how the resulting attitudinal constructs, namely 'maths disposition' and 'maths identity' change during one academic year, and between various groups of interest (e.g. gender). We conclude with a brief discussion of methodological and educational implications.

INTRODUCTION

The importance of mathematics to students' access to Science, Technology, Engineering and Mathematics (STEM) subjects in Higher education, and hence to their educational and socioeconomic life opportunities, as well as the need to promote a mathematically engaged society is well documented in literature and recent policy documents (Ofsted, 2006; Roberts, 2002; Smith, 2004). In a recent report ACME (ACME, 2009) recognises this important issue and advocates 'tackling the perceptions of mathematics" as a particularly important issue in the current economic climate, placing emphasis on the importance of mathematics as a "powerful analytical tool", with inherent "pervasiveness" and a "key workforce skill".

The paper focuses on 'attitudes to mathematics' with three particular aims: (a) to review and briefly synthesise the main points of the recent debate on the issue of 'attitudes to mathematics', (b) to present the measurement methodology we employed to capture 'attitudes to mathematics' in the context of an on-going ESRC project with secondary school students and their teachers, and (c) to report some preliminary descriptive substantive results about how this attitudinal construct changes during one academic year.

THEORETICAL PERSPECTIVE

The study of students' attitudes towards mathematics has gained considerable interest over the past 40 years. A lot of instruments (e.g. Lim & Chapman, 2013) have been proposed and used since then with a key influence the widely used Fennema-Sherman scales (Fennema & Sherman, 1977). Each of those instruments attempted to capture one of the many 'dimensions' or constructs associated with 'attitudes towards mathematics': beliefs, values, identities, engagement, affect, emotions, motivation, confidence, self-efficacy, dispositions, are only a few on the list (Ruffell, Mason, & Allen, 1998). This complexity, as well as the lack of agreement on the definition of the construct has led researchers (e.g. Watson, 2011) to recently revisit the established instruments of the 1970s and 1980s looking for alternative universal definitions or more parsimonious instruments. A useful starting point to this conceptualisation is probably Ruffell et al.'s (1998) decomposition of attitudes into three sub-components, namely cognitive, affective and conative. Their reflective analysis, as well as others that followed did not manage to reach consensus on the topic.

Despite these controversies, the study of students' attitudes and/or dispositions is very important because this may reveal key influences on their choices and decision-making and hence future engagement with STEM (Archer, Halsall, Hollingworth, & Mendick, 2005) . Previous studies had also identified a plethora of socio-cultural factors which are significant in shaping students' dispositions and choice-making in education in general, and in STEM subjects and mathematics in particular: class, gender, nationality, ethnicity, parental and peer cultures are just the beginning of the list . In our earlier work with post-secondary students we had also contributed with instruments for measuring what we called dispositions and self-efficacy in mathematics (Pampaka, et al., 2011; Pampaka et al., 2013).

Our current work, reported here, also attempts to add to this debate by a new concise instrument of students' attitudes and dispositions towards mathematics. The overall aim of this study is to understand (i) how learners' dispositions to study mathematics develop through secondary school, (ii) how mathematics pedagogies vary across different situations and contexts and (iii) how these pedagogies influence learning outcomes (including attitudinal ones).

METHODOLOGY

Project Design: The Teleprism Study

The paper is empirically based on initial findings from an on-going ESRC funded study of teaching and learning secondary mathematics in UK (www.teleprism.com). The project is designed to capture the five years of students' progression in Secondary Education (Year 7 to 11, i.e. students aged 11 to 16) in about one year of data collection: From October 2011 to December 2012. This design poses a series of methodological challenges around the combination of longitudinal and cross-sectional analyses, which go beyond the scope of this paper. The research question we seek to answer in this paper regards measuring 'dispositions' and attitudes to mathematics.

Instrumentation and Sampling

The nature and design of the study (i.e. longitudinal at school level for selection purposes) make it necessary to employ a varied sampling frame to ensure maximum coverage of the schools of England. We invited schools, drawing on various sources (including national databases), with an initial requirement for them to take part with all their Year 7 to 11 mathematics teachers and classes and be willing to follow this up at two more data collection points (hereafter DPs). In total, we approached over 2200 schools and we were able to establish collaboration with 40 of them. We note here issues around self-selection bias in this type of studies, which limits the representativeness of the achieved sample.[1]

Data collection in these schools involved a student questionnaire (at all three data points, as shown in Table 1) about students' attitudes to mathematics, confidence at various mathematical topics, future aspirations, and their perceptions of the teaching they encounter. The latter was also captured through a teacher survey administered to their mathematics teachers (twice during the course of the first academic year of the study, 2011-2012, i.e. along students' DP1 and DP2). Student questionnaires are based on different versions of the same instrument to reflect the age and level of students (i.e. 5 different Year Groups, from hereafter Y7 to Y11). Background variables and measures of students' attainment are also being collected including gender, ethnicity, language of first choice, proxies of socioeconomic status, and earlier National Curriculum level records. The various sections of the questionnaire capturing teaching and learning perceptions have been constructed and expanded based on our previous TransMaths framework (www.transmaths.org) where we validated and used instruments for students aged 16 and older (Pampaka, Kleanthous, Hutcheson, & Wake, 2011; Pampaka et al., 2013; Pampaka et al., 2012). The achieved sample size at each data point, from the participating 40 schools is summarised in Table 1, with the different completion patterns. It should be noted that some schools dropped out during the study.

Sample Description N=student numbers	**DP1** Oct - Dec 2011	**DP2** June/July 2012	**DP3** Oct – Dec 2012
Matched at all DPs	3744	3744	3744
Completed only one DP	5358	1186	2127
Completed DP1 and DP2	3051	3051	-
Completed DP1 and DP3	1172	-	1172
Completed DP2 and DP3	-	771	771
Total cross sectional sample	**13325**	**8752**	**7814**

Table 1: Sample Description [based on preliminary matching, unique cases: 17448]

For this analysis we focus on the instrument developed to capture students' mathematical attitudes, with the items, and the response format, shown in Figure 1.

[1] However we have plans in place to investigate the comparability of our sample to the national one

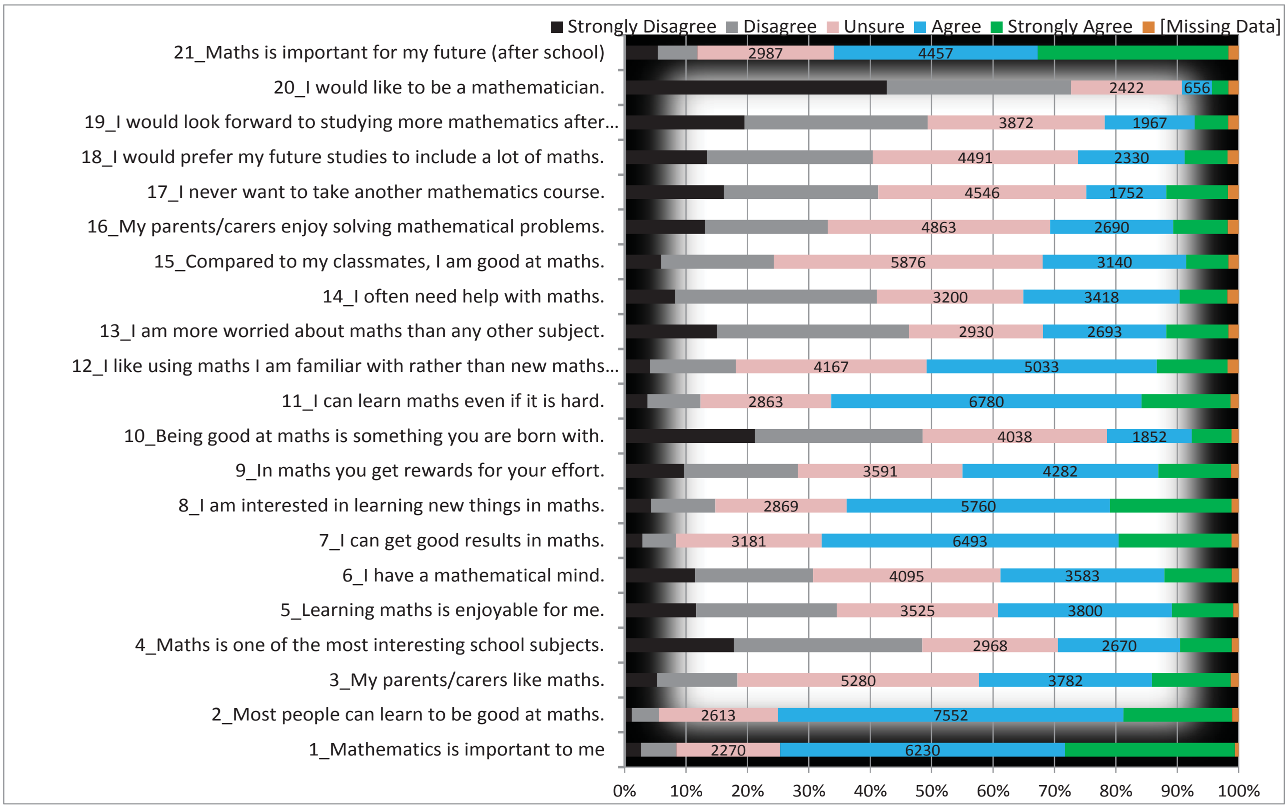

Figure 1: The items of the instrument for students' 'attitudes' towards mathematics, with the distribution of their responses at Data Point 1 (DP1, N=13325)

For the validation of the constructed measures (outlined in the next section) we draw on data from the cross sectional samples at each DP, whereas for some comparative, substantial results based on these measures, we limit analysis here to the 3744 matched cases who completed all DPs.

A measurement approach to construct validation

The validation process refers to the accumulation of evidence to support validity arguments regarding the students' disposition measures. Our psychometric analysis for this purpose is conducted within the Rasch measurement framework, following relevant proposed guidelines (Wolfe & Smith Jr., 2007) based on Messick's definitions of validity (Messick, 1989). The Rasch model is preferred because it provides the means for constructing interval measures from raw data. We have been extensively employing this approach for the validation of our constructed measures (Pampaka, et al., 2011; Pampaka, et al., 2013; Pampaka, et al., 2012). The Rasch rating scale model (using the Winsteps software) is considered the most appropriate for the scaling problems we have in this particular paper (i.e. a common Likert type scale). Our decisions about the validity of the measures are based on the following statistical indices (all these have been examined but cannot be all presented in this limited space):

- ***Item fit statistics*** to indicate how accurately the data fit the model, providing evidence in support (or not) of the unidimensionality assumption.

- ***Category Statistics*** are examined for the appropriateness of the Likert scale used and its interpretation by the respondents (i.e. communication validity).
- ***Person – item maps*** and the item difficulty hierarchy provide evidence for substantive, content and external validity.
- ***Differential Item Functioning*** (DIF) suggests potential group differentiation, which is important when an instrument is used with different groups or at different occasions (e.g. gender, year group and DP for time invariance).
- Qualitative data from interviews with students (in two case study schools) are used along the survey results, for validation, and deeper insight.

Further Statistical Modelling

Eventually, once the measures' validity is established we proceed with further statistical modelling to investigate and model change in attitudes and its association with other measures of pedagogy (Pampaka, et al., 2012) or attainment. We limit the presentation here to some descriptive results.

SELECTED RESULTS

Measuring 'attitudes' towards mathematics

As mentioned earlier, our instrument was intended to measure a general attitude in mathematics, as defined by the mixture of items. Following the measurement framework described above would provide us evidence of this hypothesis in regards to the unidimensionality of this construct. The evidence for this in the Rasch context is given by fit statistics which are local indicators of the degree to which the data is cooperating with the model's requirements. Inconsistent data (e.g. those departing from the ideal of 1) may become a source of further inquiry. For the purposes of this paper we take any number above 1.3 (of infit MnSq) as possible cause of concern, whereas infit values below 1 are considered as overfits and are not discussed. The results from our initial analysis with all the items to define a measure of 'mathematical attitudes' were not supportive for this hypothesis and operationalization: a few items were signified as misfitting (i.e. Items 10, 12, 13, 14 and 21). A unidimensionality test also suggested the existence of two dominant dimensions, with the following split of items which we explored further:

- Sub-dimension 1: Items 1, 4, 5, 8, 17, 18, 19, 20 and 21
- Sub-dimension 2: Items 2, 3, 6, 7, 11, 12, 14, 15, and 16

Separate Rasch rating scale models were performed on these two sub-dimensions, with all available data at each data point, combined together (resulting in a sample of 30000+) in order to check for DIF between DPs to ensure measure invariance over time. The fit statistics of these two measures are presented in Tables 2, for what we call mathematics disposition and Table 3, for mathematics 'identity'. For the former, two items are found to be misfitting (Item 17: I never want to take another mathematics course, and Item 21: Maths is important for my future). The coding of Item 17 was reversed for this analysis, and this might be causing its misfit. Item 21, seems to be one of the most difficult items of this measure (as indicated by its low measure value). Both are considered useful for this construct, so it was decided to keep them in the model. The psychometric properties of the second construct of 'identity' do not present any problems in regards to fit statistics.

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PT-MEASURE CORR.	PT-MEASURE EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	ITEM
1	119826	30547	-1.57	.01	.93	-8.8	.91	-9.9	.69	.69	59.3	54.0	statement1
2	82107	30418	.59	.01	.85	-9.9	.85	-9.9	.78	.73	53.8	47.6	statement4
3	91439	30454	.09	.01	.80	-9.9	.80	-9.9	.79	.73	54.2	47.6	statement5
4	109298	30395	-.93	.01	.84	-9.9	.83	-9.9	.74	.71	56.6	50.4	statement8
5	97413	30170	-.29	.01	1.48	9.9	1.72	9.9	.62	.73	47.6	48.2	statement17
6	82637	30182	.53	.01	.68	-9.9	.70	-9.9	.80	.73	59.0	47.1	statement18
7	77153	30180	.83	.01	.77	-9.9	.76	-9.9	.78	.73	57.3	47.9	statement19
8	57508	30194	2.04	.01	1.18	9.9	1.13	9.9	.66	.70	56.3	56.0	statement20
9	114163	30198	-1.28	.01	1.53	9.9	1.51	9.9	.60	.70	43.6	52.5	statement21
MEAN	92393.8	30304	.00	.01	1.01	-3.2	1.02	-3.3			54.2	50.1	
S.D.	18822.5	139.2	1.08	.00	.30	9.3	.34	9.3			5.0	3.1	

Table 2: Item fit statistics for "Mathematics Disposition"

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PT-MEASURE CORR.	PT-MEASURE EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	ITEM
1	117106	30440	-.83	.01	.97	-3.8	1.01	1.7	.48	.56	56.8	51.9	statement2
2	99364	30384	.11	.01	1.01	1.5	1.03	3.9	.61	.60	47.5	45.5	statement3
3	92493	30433	.44	.01	.85	-9.9	.86	-9.9	.75	.61	46.7	43.7	statement6
4	113759	30438	-.63	.01	.73	-9.9	.70	-9.9	.70	.57	59.8	50.1	statement7
5	110610	30295	-.49	.01	.79	-9.9	.76	-9.9	.67	.58	57.1	49.2	statement11
6	103150	30164	-.11	.01	1.30	9.9	1.40	9.9	.43	.60	45.0	46.8	statement12
7	92695	30153	.39	.01	1.26	9.9	1.43	9.9	.51	.61	40.1	43.6	statement14
8	92089	30181	.43	.01	.72	-9.9	.75	-9.9	.67	.61	53.3	43.5	statement15
9	86303	30165	.69	.01	1.22	9.9	1.26	9.9	.57	.62	39.7	42.7	statement16
MEAN	100841	30295	.00	.01	.98	-1.4	1.02	-.5			49.6	46.3	
S.D.	10319.2	122.8	.51	.00	.22	8.7	.26	8.8			7.0	3.2	

Table 3: Item fit statistics for Mathematics 'Identity'

Further investigations of DIF as well as category statistics are in support of healthy measures (these results will be provided for the interested reader at www.teleprism.com/PME2014 and will accompany the presentation).

Using the constructed measures in further analysis

The corresponding resulting scores (in logits) of the students in these measures were extracted and added in the datasets for further analysis: higher score indicate higher disposition and more mathematical 'identity'. Some descriptive results with these measures are shown next with the matched sample (N=3744), in relation to change over time, by year group and gender.

Figure 2 shows students dropping mathematical attitudes as well as some gender and year group differences. It should be noted that Year 11 was excluded from this analysis due to the limited matching sample (<100). The other sample sizes are as follows: Y7=1249, Y8=856, Y9=734 Y10=742.

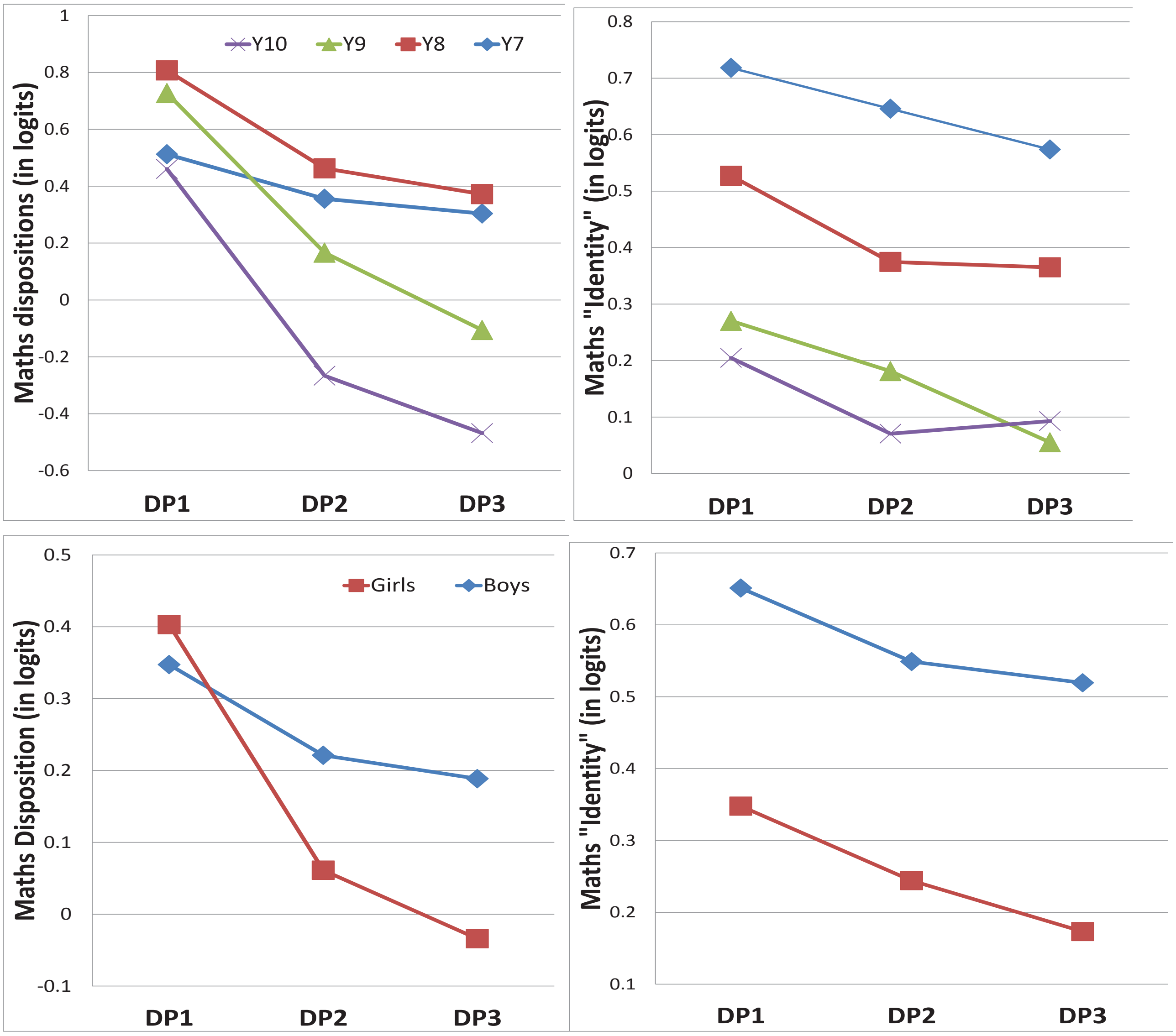

Figure 2: Changes in maths disposition (left plots) and 'identity' (right plots), by year group (top plots) and gender (bottom plots).

CONCLUDING REMARKS

Our results in regards to the dimensionality of mathematical attitudes are in agreement with earlier conceptualisations (Ruffell, et al., 1998) of attitudes as a multidimensional construct that could be decomposed into the affective, conative and cognitive components: Our 'identity' measure is constructed based on 'expressions of feelings towards mathematics, thus is closely related to the affective component. Disposition is constructed based on expressions of behavioural intention, thus it corresponds to the conative component. To this we should add that our instruments include a contextualised self-efficacy instrument, which we believe is linked to the cognitive aspect, and we intend to test in the near future.

Results with these measures (Figure 2) are in support of previous findings in regards to students' dropping dispositions and engagement with mathematics (e.g. Pampaka, et al., 2012). However, further analysis needs to be performed to account for school effects (multilevel modelling) and associate with other measures of interests such as pedagogical practices in mathematics.

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